1978 Q10 (BOUNDS OF INTEGRATION)

(i) $\frac{dy}{dx} = \sqrt{1-y^2}$, $y=0, x=1$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{1}$

$\Rightarrow \sin^{-1}\frac{y}{1} = x + A$

$\Rightarrow \sin^{-1} 0 = 1 + A$

$\Rightarrow \sin^{-1} y = x - 1.$

$\Rightarrow \quad y = \sin(x-1)$

(ii) $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$, $\frac{dy}{dx} = 1$, $x = \frac{1}{2}$ both when $y=1$.

$\Rightarrow \frac{dp}{dx} = -\frac{1}{y^3}$ $\boxed{p \equiv \frac{dp}{dx}}$

$\Rightarrow p\frac{dP}{dy} = -\frac{1}{y^3}$

$\Rightarrow \int p\,dp = \int -\frac{1}{y^3}\,dy$

$\Rightarrow \frac{p^2}{2} = \frac{1}{2y^2} + C$

// $\Rightarrow \frac{dy}{dx} = p = 1$ when $y = 1$

$\Rightarrow \frac{1}{2} = \frac{1}{2} + C$

$\Rightarrow. \ C = 0$ //

$\Rightarrow \quad p^2 = \frac{1}{y^2}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{y}$ (why take + option?)

$\Rightarrow \int y\,dy = \int dx + D$

$\Rightarrow. \ \frac{y^2}{2} = x + D$

// $x = \frac{1}{2}$, $y = 1 \Rightarrow.$

$\frac{1}{2} = \frac{1}{2} + D$

$\Rightarrow \ D = 0$ //

$\Rightarrow \quad y^2 = 2x \Rightarrow \boxed{y = \sqrt{2x}}$

(iii) + positive

O — distance d — P (starts here)

position

$v = 0.$

Force: $F = \frac{2m}{x^5}$

Particle starts from P.

Distance $|OP| = d$

N II $\Rightarrow \frac{2m}{x^5} = m\,a$

$\Rightarrow \frac{2}{x^5} = \frac{dv}{dt}.$

or $\frac{2}{x^5} = v\frac{dv}{dx}.$ (E)

Bounds of Integration.

P: $x = d$, $v = 0$ — let $x = S$, $v = V$

[a typical point]

(E) $\Rightarrow \int_d^S \frac{2}{x^5}\,dx = \int_0^V v\,dv$

$\Rightarrow -\frac{1}{2x^4}\Big|_d^S = \frac{v^2}{2}\Big|_0^V$

$\Rightarrow -\frac{1}{S^4} + \frac{1}{d^4} = V^2$

$\Rightarrow. \ V = \sqrt{\frac{1}{d^4} - \frac{1}{S^4}}$

speed at a typical distance S from the origin.

As $S \to \infty$ $\frac{1}{S^4} \to 0$

$\Rightarrow V \to \sqrt{\frac{1}{d^4}} = \frac{1}{d^2}$

$V \to \frac{1}{d^2}$ qed.